UNIT II

Electromagnetic Potentials : We can reduce the no. of Maxwell's eqn by use of electromagnetic potentials.

Magnetic Potential :- Magnetic field can be represented by Induction lines or lines of force and the associated physical quantity is magnetic field B.

The other way to represent the field is potential. To understand the concept of magnetic potential we understand the divergence and curl of B. First we show, div B is always zero and in specific conditions curl B is also zero (generally curl $B \neq 0$). For this we include a potential which is called scalar potential. But generally curl $B \neq 0$, so we use another method to write B in terms of potential → vector potential.

(i) Divergence of $\vec{B}$

From Biot-savart law

$$B = \frac{\mu_0 i}{4\pi}\int \frac{d\vec{l}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}\times\vec{r}}{r^3}dV \quad \text{--- (1)}$$

$j \rightarrow$ current density vector

$\vec{i} = \int_S \vec{j}\cdot ds$, $ds \rightarrow$ small area enclosing volume dV.

$$i\vec{dl} = j\,ds\,dl = j\,dV$$

From eqn (1)

$$\text{div}\,\vec{B} = \vec{\nabla}\cdot\vec{B} = \vec{\nabla}\cdot\frac{\mu_0}{4\pi}\int_V \frac{\vec{j}\times\vec{r}}{r^3}\,dV$$

$$\Rightarrow \vec{\nabla}\cdot\vec{B} = \frac{\mu_0}{4\pi}\int_V \vec{\nabla}\cdot\left[\frac{\vec{j}\times\vec{r}}{r^3}\right]dV \quad \text{---}(2)$$

Using vector identity

$$\vec{\nabla}\cdot(\vec{a}\times\vec{b}) = \vec{b}\cdot(\vec{\nabla}\times\vec{a}) - \vec{a}\cdot(\vec{\nabla}\times\vec{b})$$

if $a = \vec{j}$ and $\vec{b} = \frac{\vec{r}}{r^3}$

$$\vec{\nabla}\cdot\left[\vec{j}\times\frac{\vec{r}}{r^3}\right] = \frac{\vec{r}}{r^3}\cdot(\vec{\nabla}\times\vec{j}) - \vec{j}\cdot\left(\vec{\nabla}\times\frac{\vec{r}}{r^3}\right)$$

Since $j \to$ function of source point and $\vec{\nabla} \to$ function of field point $\to$ first term of above eqn is zero

and j is also constant so $\vec{\nabla}\cdot\vec{j} = 0$

from second step

$$\vec{\nabla}\times\frac{\vec{r}}{r^3} = \vec{\nabla}\times\left[-\vec{\nabla}\left(\frac{1}{r}\right)\right]$$

Curl of gradient $= \vec{\nabla}\times\vec{\nabla}\left(\frac{1}{r}\right) = (-)\,\text{curl grad}\left(\frac{1}{r}\right)$

The ~~gradient of curl~~ is always zero for any function

So from eqn (2)

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{---}(3)$$

$\to$ One of the Maxwell eqn.

Using the Gauss divergence theorem

$$\phi_B = \int_S \vec{B}\cdot\vec{ds} = \int_V (\vec{\nabla}\cdot\vec{B})\,dV$$

$\because \vec{\nabla}\cdot\vec{B} = 0$ always $\to$ magnetic fields lines are in the form of closed loop.

Magnetic monopole does not exist.

(ii) Curl of B : From Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int_s \vec{J} \cdot d\vec{s}$$

From stokes's theorem

$$\int \text{curl} \vec{B} \cdot d\vec{s} = \mu_0 \int \vec{J} \cdot d\vec{s}$$

$$\Rightarrow \quad \text{curl} \vec{B} = \mu_0 \vec{J} \quad \text{---(4)}$$ Differential eq^n of Ampere's law.

This result is different from the electrostatic relation for electric field $\text{curl} \vec{E} = 0$ or $\oint \vec{E} \cdot d\vec{l} = 0$.

The magnetostatic field is non-conservative.

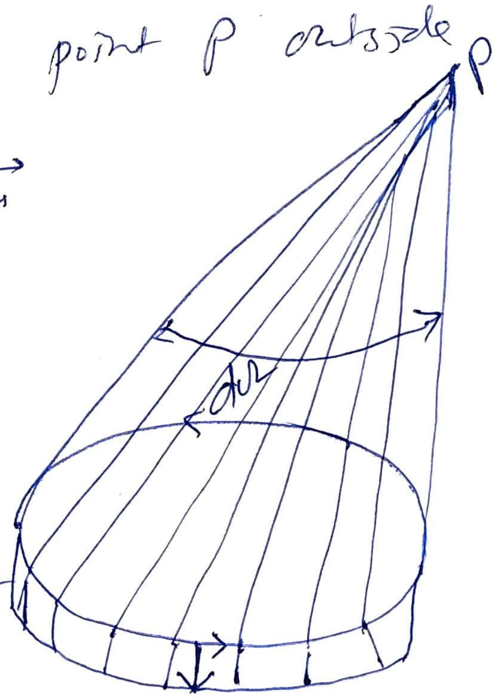
for a current carrying conductor $\text{curl} \vec{B} \neq 0$.

but ~~extra~~ $\text{curl} \vec{B} = 0$ in empty space.

(iii) Scalar Potential :- We take a close loop of wire, in which non-changing current is flowing. The solid angle at point P outside loop is Ω.

If P point is displaced by $d\vec{u}$ then change in solid angle will be $d\Omega$.

This change is same as the change $d\Omega$, which happens due to displacement of loop by $-d\vec{u}$



In such a displacement an element dl of the circuit sweeps an area $-d\vec{u} \times d\vec{l}$. The angle subtended ~~at~~ at P by this area is

$$d\Omega = = -d\vec{u} \cdot \oint \frac{d\vec{l} \times \vec{r}}{r^3}$$

change $d\Omega$ can also be written as $\text{grad}\,\Omega \cdot d\vec{u}$.

So $$d\vec{u} \cdot \text{grad}\,\Omega = -d\vec{u} \cdot \int \frac{d\vec{l} \times \vec{r}}{r^3}$$

or $$-\text{grad}\,\Omega = \int \frac{d\vec{l} \times \vec{r}}{r^3}$$

Comparing with eq^n (1)

$$\vec{B} = -\frac{\mu_0 i}{4\pi}\,\text{grad}\,\Omega$$

or $$\vec{B} = (-)\,\text{grad}\,\phi_m \quad \text{---(5)}$$

where $$\phi_m = \frac{\mu_0 i}{4\pi}\,\Omega \quad \text{---(6)}$$

$\phi \rightarrow$ scalar potential, the negative gradient of ϕ gives magnetic field $\vec{B}$.

If $B \rightarrow$ is obtained from scalar potential, then curl $\vec{B} = 0$, because gradient of curl is zero.

But generally curl $\vec{B} \neq 0$ $\longrightarrow$ so we can not take B derived from scalar potential, we take new concept of vector potential.

For electrostatic field, we can represent $\vec{E}$ ~~in the~~ as the negative ~~grad~~ gradient of scalar potential ($\vec{E} = -\text{grad}\,V$) $\Rightarrow$ $\vec{E} = -\nabla V$, but then curl $\vec{E} = 0$ and $\oint \vec{E} \cdot d\vec{l} = 0$, the value of scalar potential of electric field on any point is ~~zero~~ unique but this is not true for magnetic scalar potential, because from Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 i$